



$$s = k \cdot (w \cdot d^2)$$

$$s = k \cdot w (144 - w^2)$$

$$s = k (144w - w^3)$$

$$12^2 = w^2 + d^2 \quad s' = k (144 - 3w^2) = 0$$

$$d^2 = 144 - w^2$$

$$s'' = k(-6w) < 0 \quad \sqrt{\frac{144}{3}} = w$$

max at $\sqrt{\frac{144}{3}} = w$

(--)

Oct 13-8:58 AM

4.4c Modeling and Optimization

Examples from Economics

Maximum Profit: If there is a maximum profit, it occurs when marginal revenue = marginal cost

$$P = R - C$$

$$P' = R' - C' = 0$$

$$R' = C'$$

Oct 25-8:00 AM

Suppose $r(x) = 9x$ and $c(x) = x^3 - 6x^2 + 15x$, where x represents 1000's of units. Is there a production level that maximizes profit? If so, what is it?

$$R' = C'$$

$$9 = 3x^2 - 12x + 15$$

$$x = .585786$$

$$x = 3.41421$$

max profit at

$$P = R - C$$

$$P' = 9 - (3x^2 - 12x + 15) = 0$$

$$P'' = -6x + 12$$

$$P''(.585786) > 0 \quad (+)$$

$$P''(3.41421) < 0 \quad (-)$$

Oct 25-8:05 AM

$$\text{average cost} = \frac{c(x)}{x}$$

Minimum Average Cost: If there is a minimum average cost, it occurs when average cost = marginal cost

$$AC = \frac{c(x)}{x}$$

$$AC' = \frac{x \cdot c'(x) - c(x) \cdot 1}{x^2} = 0$$

$$x \cdot c'(x) = \frac{c(x)}{x}$$

Oct 25-8:06 AM

Suppose $c(x) = x^3 - 6x^2 + 15x$, where x represents 1000's of units. Is there a production level that minimizes average cost? If so, what is it?

$$AC = \frac{c(x)}{x} = \frac{x^3 - 6x^2 + 15x}{x} = x^2 - 6x + 15$$

$$AC' = 2x - 6 = 0$$

$$\text{min at } x = 3$$

$$AC'' = 2 > 0$$



$$3x^2 - 12x + 15 = x^2 - 6x + 15$$

$$2x^2 - 6x = 0$$

$$2x(x-3) = 0$$

$$x = 0 \quad x = 3$$

Oct 25-8:08 AM

4.4

27. Tour service

\$200/person or discount if more than 50 people

discount = \$2 off for each person beyond 50

6000 fixed cost

\$32/person additional costs

what if 60 people discount = 20 off

charge \$180/person
200 - 2(x-50)

$$P = 180 \cdot 60 - (6000 + 32 \cdot 60)$$

Oct 13-10:24 AM